Standard Deviation

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

This equation is used to find the average deviation of values from the mean of those values. The formula above can easily be remembered by the phrase "the square root of the mean of the squares minus the square of the mean". Σx^2 means we add the squares of all of our values.

For example, let there be a list of values, and we want to find the standard deviation:

$1, 2, 2, 2, 4, 5, 6, 7, 7$

The mean, often written as $\bar{x}$, can be calculated by the equation $\bar{x} = \frac{\Sigma x}{n}$ so the mean is found by summing all the values and dividing them by the number of values.

$$\bar{x} = \frac{1+2+2+2+4+5+6+7+7}{9}$$

$$\bar{x} = 4$$

We can use this as part of the calculation for the standard deviation. As

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{x}{n}\right)^2}$$

And $\bar{x} = \frac{x}{n}$ we can therefore say that

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$$

If we input the squares of the above values

$$\sigma = \sqrt{\frac{1^2+2^2+2^2+2^2+4^2+5^2+6^2+7^2+7^2}{9} - (4)^2}$$

$$\sigma = \sqrt{\frac{188}{9} - 16}$$

$$\sigma = 2.211 \ldots$$

Proof

If we were to try to find the average deviation (or difference) from the mean, we would most probably subtract each value mean, then divide by the number of values.

$$\frac{1}{n}\sum(\bar{x} - x)$$

However this will always equal zero, because

$$\frac{1}{n}\sum(\bar{x}-x)=\frac{1}{n}\sum\bar{x}-\frac{1}{n}\sum x$$

Which can be written

$$\frac{1}{n}\sum(\bar{x}-x)=\frac{\Sigma\bar{x}}{n}-\frac{\Sigma x}{n}$$

We already know $\bar{x}=\frac{\Sigma x}{n}$

$$\sum(\bar{x}-x)=\frac{\Sigma\bar{x}}{n}-\bar{x}$$

And if we add up n lots of $\bar{x}$

$$\sum(\bar{x}-x)=\frac{\Sigma\bar{x}}{n}-\bar{x}=\frac{n\bar{x}}{n}-\bar{x}=0$$

We can avoid this by finding the absolute values of the differences between the mean and the values, so for values larger thar

$$\frac{1}{n}\sum|\bar{x}-x|$$ [1]

However, it can be inconvenient to try to work with finding absolute values. We can get round this by squaring the differenc
4 and $(+2)^2=4$).

$$\frac{1}{n}\sum(\bar{x}-x)^2$$

We can expand this, though, to get

$$=\frac{1}{n}\left(\sum(\bar{x})^2-\sum 2\bar{x}x+\sum x^2\right)$$

We can re-write this in the form

$$=\frac{1}{n}\left(\sum x^2+\sum(\bar{x})^2-\sum 2\bar{x}x\right)$$

As before, $\Sigma\bar{x}=n\bar{x}$ and $\Sigma x=n\bar{x}$

$$=\frac{1}{n}\left(\sum x^2+n(\bar{x})^2-2n(\bar{x})^2\right)$$

Simplifying

$$=\frac{1}{n}\left(\sum x^2-n(\bar{x})^2\right)$$

Expanding the bracket

$$=\frac{\sum x^2}{n}-(\bar{x})^2$$

What we have here is called the variance. This is not the standard deviation, as one problem with squaring numbers is that alt
− root to counteract this problem.
Thus

$$\sigma=\sqrt{\frac{\Sigma x^2}{n}-(\bar{x})^2}$$

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[1] These straight lines represent the modulus function, and finds the absolute value of numbers, so 'ignores' minus signs, e.g. $|-2|=2$ and $|+2|=2$. This will stop negatives from cancelling out positives to get a standard deviation of zero.

References

Turner, L. K. (1976). *Advanced Mathematics – Book One*. London: Longman. pp.260-263.